Analysis Of Complementary Perfect Domination Number and connectivity Of Regular Graphs

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Abstract **-** In this paper authors describe domination number of regular graphs as well as complimentary perfect domination number and induced complimentary perfect domination number, denoted by cpd and icpd. Let G(v,e) be a graph with n vertices and e edges then these are denoted by $X_{cp}(G)$.and $X_{icp}(G)$.In this paper author describe, How to calculate $X_{cp}(G)$.and $X_{icp}(G)$ of regular graphs. Authors characterize 2 regular graphs with X_{icp} and 3 regular graphs with X_{icp} and describe an upper limit for number of vertices in d-regular graph. In the end of the paper authors characterize all the d-regular graphs with $X_{icp}(G)$ and practical utilization of cpd and ipcd.

Keywords **- Domination number, representation of two graphs with one, cpd and ipcd of regular graph, d-regular.**

INTRODUCTION

In this modern age of science and pacific root of computer life, so many scholars are doing hard work in the domination theory. In this paper authors plane a structure of traffic control system with regular graphs and explain how it is used. because more than lacks Rs. are spent on the flyover of a crossing.

Let G(v,e) be a regular graph with n vertices and e edges that means Graph is finite, simple undirected. The number of vertices is denoted by V_1, V_2, V_3,. . . . . ,V_n. A lot of work has been done on domination sets and domination number. Different type of domination sets and domination number has been introduced and studied by several authors. [1][4][5][7][10] . Paulraj Joseph and Arumugam S have found the relationship between domination number and connectivity in graph as well as domination number and coloring in graphs.

In the same way Paulraj J, Joseph and Mehadevan [11] have introduced the concept of complimentary perfect domination number but they fail to calculate cpd and icpd of regular graphs.

Avadayappan and Senthil Kumar have proved that for any graph G and a vertex V of G. $X_{cp}(G_m(v)) = X_{cp}(G) + m$, where $G_m(v)$ is the graph obtained from G by identification of the center vertex of a star graph $K_{i,m}$ at a vertex V.

Domination Sets:-

A subset S of vertices is known domination set of G if every vertex in V-S is adjacent to at least one vertex in S.

Domination Number: -

Domination number of a graph G is the minimum cardinality taken over all dominating set in G and denoted by X(G).

Complementary Perfect Domination Number: -

The complimentary perfect domination number of graph G is the minimum cardinality taken over all complementary perfect domination sets in g and denoted by $X_{cp}(G)$.

Induced Perfect Domination Number: -

The minimum cardinality taken over all the induced perfect Dominaiton sets of G is known as induced complementary perfect domination number of G and denoted by $X_{icp}(G)$. An induced complementary perfect domination set of a graph G (icpd-set) is a domination set of G such that induced sub graph(v-s) has only independent edges. Any ipcd-set with X_{icp} elements is known as X_{icp} -set of G.

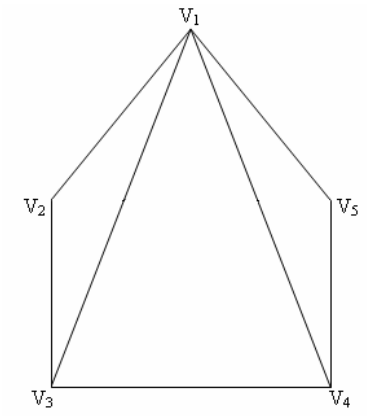


Fig1.1

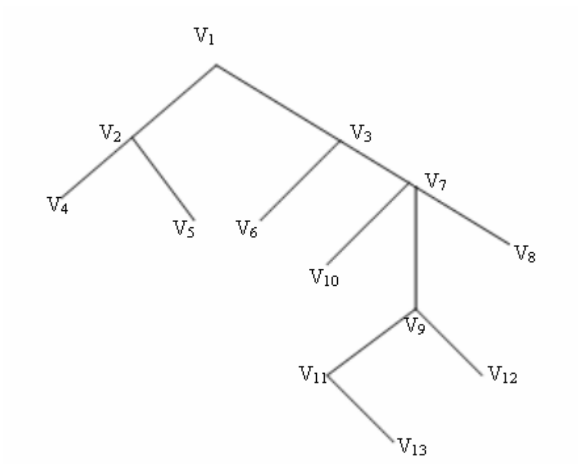


Fig 1.2

In fig 1.1 $X_{cp}(G)=1$ and $X_{icp}=3$

Here S = $\{V_1\}$ is the X_{cp}-set and S = $\{V_1, V_2, V_3\}$ is a X_{icp}-set as well as fig 1.2 has X_{cp}= 8 and $X_{icp}=9$. Here S = $\{V_2, V_5, V_6, V_8, V_{10}, V_{12}, V_{13}\}$ is X_{cp}- set and S = $\{V_3, V_5, V_6, V_8, V_{10}, V_{11}, V_{12}, V_{13}\}$ is a X_{icp}-set.

NOTATIONS

G(v,e) → A graph with n vertices and e edges.
n-regular graph → A graph G is said to be n –regular if degree of Each vertex is n.
d(v) → Degree of vertex V in G.
Δ → Maximum degree in G.
$X_{cp}(G)$ → Complimentary perfect domination number of G.
$X_{icp}(G)$ → Induced complimentary perfect domination number of G.
[x] → Greatest number, which is at least x.
K_n → Complete graph with n vertices.
C_n → Cyclic on n vertices.
<S> or (S) → Induced sub graph of G.
r → Chromatic number of a graph.
$K_{3,3}$ → Bipartite graph.

Assumptions / Observation

1. For any Graph, $n \geq 2$ $X_{icp}(G)=n$ iff G is a star.
2. $X_{icp}(G)=1$ $\Leftrightarrow$G is isomorphic to $K_1 \vee mK_2$, $m \geq 1$
3. Any Icpd – set of G must contain all the Pendant vertex of G.
4. If G is not star then $\left[\frac{n}{\Delta+1}\right] \leq X_{icp} \leq n-2$
5. $X_{icp}(K_n)=n-2 \quad \forall \quad n \geq 3$

Complementary Perfect Domination Number and connectivity Of Regular Graphs

This section authors try to fill some gap by solving some theorem and its characterization, all cycles of 2-regular graph with $X_{cp}=\alpha$ and 3-regular graphs with $X_{icp}=\alpha=2$

<u>Theorem 1.1</u>:-

G is 2-regular graph with $X_{cp}(G)=r \Leftrightarrow G \cong c_n, n=4,5,6,7,9$

<u>Proof</u>: - Given that G is 2-regular i.e. 2-regular is a cycle.

$\Rightarrow$ Cycle may be even or odd
Even cycle have $r=2$ and odd cycle have $r=3$.
$\Rightarrow$ $X_{cp}=r$
$\Rightarrow$ X_{cp} = 2 or 3

Now let $X_{cp}=r=2$ then

$$2=X_{cp} \geq \left[\frac{n}{\Delta+1}\right]=\left[\frac{n}{3}\right]$$

Therefore $n \leq 6$
Hence $G \cong C_4$ or C_6

In the same way

$X_{cp} = r = 3$ Then $3 = V_{cp} \geq \left[\frac{n}{\Delta+1}\right] = \left[\frac{n}{3}\right]$

$\Rightarrow$ $P \leq 9$ And $G \cong C_5$ or C_7 or C_9

Conversely,

$$G \cong C_n, n = 4,5,6,7,9$$

$\Rightarrow$ There exist some circuits of even and odd length.

$\Rightarrow$ G is 2-regular with $X_{cp} = r$

<u>Theorem 1.2</u>: -

G is a 3 regular graph with $X_{cp} = r = 2$ if and only if $G \cong K_{3,3}$

<u>Proof</u>: - First assuming that G is 3-regular graph with $X_{cp} = r = 2$ and prove that $G \cong K_{3,3}$

G be 3-regular graph with $X_{cp} = r = 2$ than $2 = X_{cp} \geq \left[\frac{n}{\Delta+1}\right] = \left[\frac{n}{3}\right]$

$\Rightarrow$ $n \leq 8$

$\Rightarrow$ but G is 3-regular

$\Rightarrow$ n = 4, 5, 6 or 8

$\Rightarrow$ $n \neq 4$

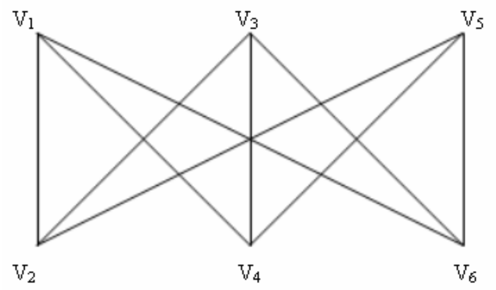
Because K_4 is the only 3-regular graph on 4-vertices of which r = 4.

$\Rightarrow$ n = 6 or 8 i.e. there are 2 case.

<u>Case I</u>: - n = 6

Let $V_1, V_2, V_3, V_4, V_5, V_6$ be the six vertices of G. Let S={V_1, V_2} be X_{cp}- set then <V-S> should have two independent edges say $V_3 V_4$ and $V_5 V_6$. Since r = 2, G has no triangle and hence both V_3 and V_4 or V_5 and V_6 cannot be adjacent to V_1 i.e. V_1 is adjacent to at most 2 non-adjacent vertices of V.S. but G is 3-regular with r =2 therefore V_1 is adjacent to V_2 and to exactly non-adjacent vertices of V-S say V_4 and V_6.

Similarly V_2 is Adjacent to V_1 and to exactly two non-adjacent vertices of V-S say V_3, V_5. Also a vertex in <V-S> cannot be adjacent to both V_1 and V_2. Keeping all above calculation the resulting graph is nothing but $K_{3,3}$ as shown in fig.



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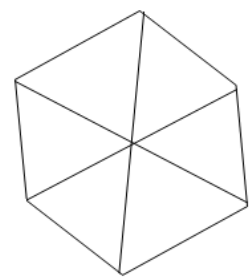


<u>Case II</u>: - If n = 8

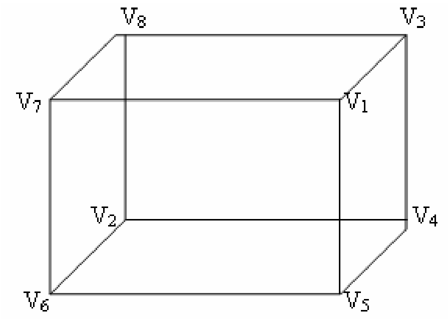
Let $V_n \in G \;\; \forall \;\; n = 1,2,\ldots.,8$ with $X_{cp} = 2$. Let S = {V_1, V_2} be a $X_{cp} - set$. As G is A 3-reglarar graph and S is dominating set in V-S, V_1 and V_2 are adjacent to exactly three different vertices in V-S.

Let $R(V_1) = \{V_3, V_5, V_7\}$ and $R(V_2) = \{V_4, V_6, V_7\}$. Since r = 2, G has no triangle and V_3, V_5, V_7 are independent. Similarly V_4, V_6, V_8 are independent.

Now <V-S> is 2-regular bipartite graph with (U,V) Where U={V_3, V_5, V_7} and V={V_4, V_6, V_7, V_8}$\Rightarrow$<V-S> is 2-regular with six vertices.

$$\Rightarrow \; < V - S > \cong C_6$$

Here resulting graph shown as



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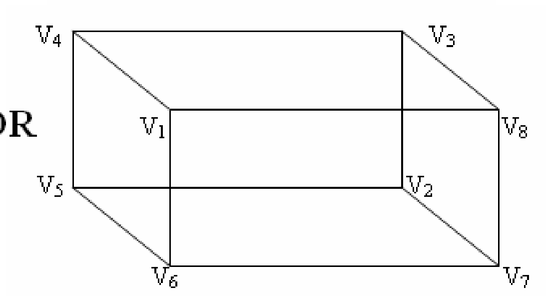


conversely, by theorem we know that r=3 of $K_{3,3}$

r = m for $K_{3,3}$, m > b

Induced Perfect Domination Number and connectivity Of Regular Graph.

In this section, Author characterizes all the d-regular graphs with minimum possible induced complimentary perfect domination number.

<u>Theorem 1.3</u>:-

If G is a d-regular graph then $X_{icp}(G) \geq d-1$ and $n \leq X_{icp}\left(\frac{2d-1}{d-1}\right)$

<u>Proof</u>: - Let S be a set $X_{icp}-set$ in G. then all the vertices in <V-S> have degree 1.

(From definition of X_{icp})

Since the Graph is d-regular every vertex in <V-S> is adjacent to d-1 vertices in S.

Therefore $X_{icp} \geq d-1$. In 1998 [8] Slater P.J. proof that $dX_{icp} \geq (n - X_{icp})(d-1) = nd - n - dX_{icp} + X_{icp}$

i.e. $X_{icp} \geq n\left(\frac{d-1}{2d-1}\right)$

or $n \leq X_{icp}\left(\frac{2d-1}{d-1}\right)$

Let Gi denote any i-regular on d-1 vertices where $d \geq 2$ and $2 \leq i \leq d-2$, both d and i have same property.

When i=0,1,2 and d-1, then $G_i \cong K^c_{d-1}, \left(\frac{d-1}{2}\right)K_2, C_{d-1}$ and K_{d-1} respectively.

<u>Theorem 1.4</u>: -

A d-regular graph of a graph G with $X_{icp} = d-1$ iff $G \cong G_i V\left(\frac{d-1}{2}\right)k_2$ where $0 \leq i \leq d-2$

<u>Proof</u>: - Let G be A d-regular graph with $X_{icp} = d-1$ and prove that $G \cong G_i V\left(\frac{d-1}{2}\right)k_2$ given that G is a d-regular with $X_{icp} = d-1$

$\Rightarrow$ <S> has d-1 vertices and <V-S> has only independent edges.

By Theorem 1.3, $n \leq 2d-1$ and sumce G is d-regular $n \geq d+1$

$\Rightarrow d+1 \leq n \leq 2d-1$

If <V-S> has $\left(\frac{d-i}{2}\right)$ edges. Where $0 \leq i \leq d-2$ and both i & d have same property.

- $\Leftrightarrow$ <V-S> is 1-regular graph with d-I vertices.
- $\Leftrightarrow$ G is d-regular
  (Every vertex in <V-S) is adjacent to all the d-1 vertices in <S>)
- $\Leftrightarrow$ Every vertex in S is adjacent to d-i vertices in V-S.
- $\Leftrightarrow$ Every vertex in S is adjacent to exactly I vertices in S.
- $\Leftrightarrow$ <S> is i-regular and S has d-1 vertices.
- $\Leftrightarrow < S > \cong G_i$
- $\Leftrightarrow < S > \cong G_i \vee \left(\frac{d-i}{2}\right)K_2$

<u>Illustration</u>:-

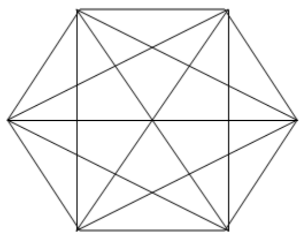
1) From these theorem and calculations authors conclude that

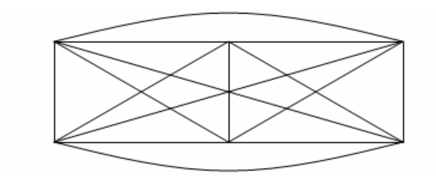
2-regular graph with $X_{icp} = 1$

3-regular graph with $X_{icp} = 2$

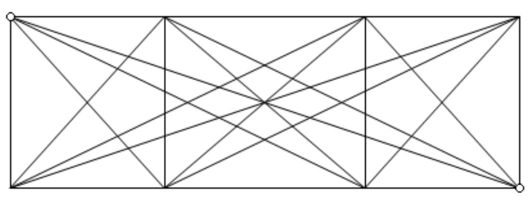
4-regular graph with $X_{icp} = 3$

5-regular graph with $X_{icp} = 4$



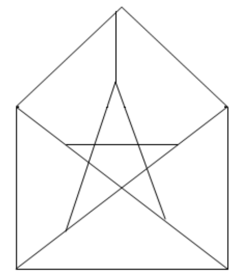
⇔ 

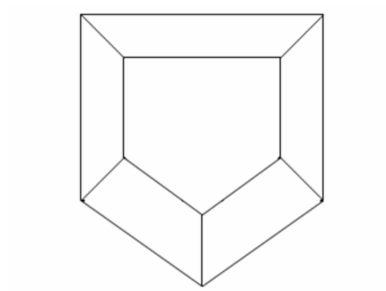
5-regular graph or K_6



5-regular or $2K_2 \vee 2K_2$

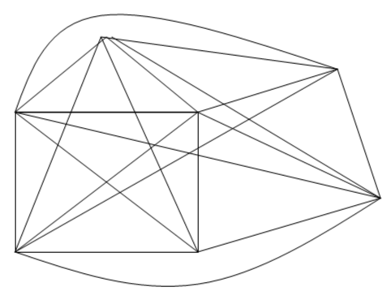
2) Theorem 1.2 and Theorem 1.4 gives the help of detection of planarity of Peterson graph.

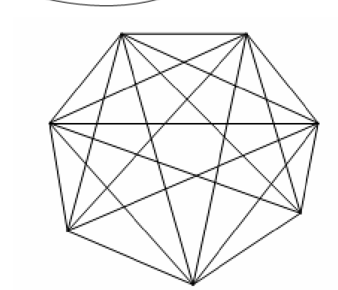




3-regular graph with n=10 is non planner
It is also a 3-regular graph but not Peterson graph

3) 6-regular Graph with $\chi_{icp} = 5$



⇔ 

$K_5 \vee K_2$
K_7

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